A New Method For Active Noise Control Systems With Online Acoustic–Feedback–Path Modeling

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Abstract—This paper proposes a new method for online modeling of the acoustic feedback path in active noise control (ANC) systems. The proposed method comprises three adaptive filters: 1) the FxLMS algorithm based noise control filter, 2) a variable step size (VSS) LMS algorithm based feedback path modeling (FBPM) filter, and 3) LMS algorithm based adaptive noise cancelation (ADNC) filter. The ADNC filter removes the disturbance from the desired response of FBPM filter. The disturbance signal in FBPM filter is thus decreasing in nature, (ideally) converging to zero. This gives basic idea for the VSS LMS algorithm for FBPM filter: a small step size is used initially and later its value is increased accordingly. The computer simulations show that the proposed method can reduce FBPM error at a faster convergence rate than the existing methods.

I. INTRODUCTION

A schematic diagram of a single channel feedforward active noise control (ANC) systems [1], [2] is shown in Fig. 1. Here P(z) is the primary path between the noise source and the error microphone, S(z) is the secondary path between canceling loudspeaker and error microphone, and F(z) is the feedback path from canceling loudspeaker to the reference microphone. The ANC system uses the reference microphone to pick up the reference noise x(n), processes this input with an adaptive filter to generate an antinoise y(n) to cancel primary noise acoustically in the duct, and uses an error microphone to measure the error e(n) and to update the adaptive filter coefficients. Unfortunately, a loudspeaker on a duct wall will generate the antinoise signal propagating both upstream and downstream. Therefore, the antinoise output to the loudspeaker not only cancels noise downstream, but also radiates upstream to the reference microphone, resulting in a corrupted reference signal x(n). This coupling of acoustic waves from secondary loudspeaker to the reference microphone is called acoustic feedback.

Consider Fig. 2, which is a block diagram of the ANC system shown in Fig. 1. Here the filtered-x LMS (FxLMS) algorithm [3], [4] is used to adapt the ANC adaptive filter W(z). Assuming that the feedback path modeling (FBPM) filter F(z) is not present, the error signal z-transform is expressed as

\[
E(z) = P(z)R(z) - S(z)Y(z) = P(z)R(z) - S(z)\frac{W(z)R(z)}{1 - W(z)F(z)}.
\]  

(1)

The convergence of \(W(z)\) means (ideally) \(E(z) = 0\). This requires \(W(z)\) to converge to the following solution:

\[
W^o(z) = \frac{P(z)}{S(z) + P(z)F(z)}.
\]  

(2)

This simple analysis shows that due to acoustic feedback the ANC system will be unstable, if the coefficients of \(W(z)\) are large enough so that \(W(z)F(z) = 1\) at some frequency. Furthermore, the \(W(z)\) may not converge to the optimal solution \(P(z)/S(z)\).

Broadly speaking there are two classes of methods for acoustic feedback compensation [1]: 1) To use directional microphones and loudspeakers and/or to place the components so that the acoustic feedback can be avoided. These techniques are expensive and often the performance is quite limited, 2) To process the reference signal, so that the feedback component

Fig. 1. Schematic diagram of a single-channel feedforward ANC system.

Fig. 2. Block diagram of ANC system of Fig. 1. with fixed feedback neutralization.

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is reduced if not removed. It includes methods based on finite-impulse-response (FIR) and infinite-impulse-response (IIR) adaptive filters. In this paper we examine only FIR-filter-based methods for acoustic feedback compensation.

The simplest FIR-filter based approach to solving the feedback problem is to use a separate FBPM filter with in the controller, as shown in Fig. 2. This electrical model of the feedback path is driven by the secondary source control signal and its output is subtracted from the reference sensor signal. The FBPM filter, \( \hat{F}(z) \), may be obtained offline prior to the operation of ANC system when the reference noise \( x(n) \) does not exist. In many practical cases, however, \( x(n) \) always exists, and \( F(z) \) may be time varying. For these cases, online modeling of \( F(z) \) is needed to ensure the convergence of the FXLMS algorithm for ANC systems.

In [5], we have proposed a method for acoustic FBPM during online operation of ANC systems. This method uses three adaptive filters; a noise control filter, a feedback path modeling (FBPM) filter, and an adaptive noise cancelation (ADNC) filter. The objective of ADNC filter is to remove the disturbance from the desired response of FBPM filter. Here we modify this method by incorporating a variable step size (VSS) LMS algorithm with the FBPM filter \( \hat{F}(z) \). The step size is varied in accordance with the disturbance in the desired response of \( \hat{F}(z) \). The desired response of \( \hat{F}(z) \) is corrupted by a disturbance signal, which is initially very large and converges to (ideally) zero. Hence a small step size is used initially, and later its value is increased accordingly.

The organization of the paper is as follows. Section 2 presents an overview of the existing methods for online FBPM in ANC systems, and describes the proposed method. Section 3 gives simulation results, and Section 4 concludes the paper.

II. ANC SYSTEMS WITH ONLINE ACOUSTIC–FEEDBACK–PATH MODELING

A. Adaptive Feedback Neutralization

The FBPM filter \( \hat{F}(z) \) can be made adaptive by using \( x(n) \), the input to \( W(z) \), as its error signal. This scheme is shown in Fig. 3 [6]. Here \( s(n) \) is the signal picked up by the reference microphone:

\[
s(n) = r(n) + y_f(n)
\]

where \( r(n) \) is the reference noise signal, \( y_f(n) = f(n) * u(n) \) is the feedback signal from the canceling loudspeaker to the reference microphone, \( f(n) \) is the impulse response of the feedback path \( F(z) \), \( y(n) \) is the output of \( W(z) \), and * denotes the linear convolution. The reference signal \( x(n) \) for \( W(z) \) is given as

\[
x(n) = s(n) - \hat{y}_f(n) = r(n) + [y_f(n) - \hat{y}_f(n)]
\]

where \( \hat{y}_f(n) \) is an estimate of \( y_f(n) \) obtained through FBPM filter:

\[
\hat{y}_f(n) = \hat{f}^T(n) y_{L_f}(n)
\]

where \( \hat{f}(n) = [\hat{f}_0(n), \hat{f}_1(n), \cdots, \hat{f}_{L_f-1}(n)]^T \) is the impulse response of the FBPM filter \( \hat{F}(z) \), \( L_f \) is the tap-weight length of FBPM filter \( \hat{F}(z) \), and \( y_{L_f}(n) = [y(n-1), y(n-2), \cdots, y(n-L_f)]^T \) is the \( L_f \)-sample output signal vector.

The output \( y(n) \) of the ANC filter \( W(z) \) is given as:

\[
y(n) = \hat{w}^T(z) x_{L_w}(n)
\]

where \( \hat{w}(n) = [w_0(n)w_1(n) \cdots w_{L_w-1}(n)]^T \) is the tap-weight vector of \( W(z) \), \( L_w \) is tap-weight length of \( W(z) \), and \( x_{L_w}(n) = [x(n)x(n-1) \cdots x(n-L_w+1)]^T \) is the \( L_w \)-sample reference signal vector.

It is clear from (3)–(6) that the desired response for \( \hat{F}(z) \), \( s(n) \), is highly correlated with the antinoise signal \( y(n) \), and hence \( \hat{F}(z) \) will continue to adapt even when the feedback is perfectly canceled. Under this condition, \( \hat{F}(z) \) attempts to incorrectly cancel the reference noise signal since it is correlated with the output of \( W(z) \). Therefore the adaptation of the feedback neutralization filter must be inhibited when the ANC system is in operation. This method, therefore, can not be used for online FBPM.

B. Kuo’s Methods

In order to overcome problems with the above method [6] and to achieve online FBPM, Kuo and Luan [7] have
proposed an additive random noise-based method as shown in Fig. 4. Here the adaptive FBPM filter is excited by a low level random (white) signal, $v(n)$, which is injected at the secondary loudspeaker for the identification of feedback path. After adaptation the weights are copied to the fixed FBPM filter taking $y(n)$ as its input. The reference signal picked up by the reference microphone, $s(n)$, is now given as:

$$s(n) = r(n) + y_f(n) + v_f(n) \quad (7)$$

where $v_f(n) = f(n) * v(n)$ is the feedback component due to the modeling signal $v(n)$. The desired response for the adaptive FBPM filter is given as:

$$s'(n) = r(n) + [y_f(n) - \hat{y}_f(n)] + v_f(n). \quad (8)$$

Here only $v_f(n)$ is required for identification and rest of components act as a disturbance signal. Due to this large disturbance signal [note that $r(n)$ will be present all the times] the convergence of the FBPM filter may be very slow.

In order to improve the convergence of $\hat{F}(z)$ in above method, Kuo proposed [8] using a predictor as a signal discrimination filter (see Fig. 5). This method assumes that the reference noise signal is narrowband, and hence can be predicted. The signal discrimination filter can predict $r(n) + [y_f(n) - \hat{y}_f(n)]$ in (8), and hence, can cancel it. Thus desired response for the adaptive FBPM filter is free of any disturbance signal, i.e. $z(n) \approx v_f(n)$. The input signal for $W(z)$ is computed by subtracting $z(n)$ from $s'(n)$:

$$x(n) = r(n) + [y_f(n) - \hat{y}_f(n)] + [v_f(n) - d_f(n)]. \quad (9)$$

Assuming that the adaptive filters $H(z)$, and $\hat{F}(z)$ are converged: $z(n) \approx v_f(n)$, $\hat{y}_f(n) \approx y_f(n)$ and $d_f(n) \approx v_f(n)$. Thus input to $W(z)$ is $x(n) \approx r(n)$, and is free of any noise. This method [8] improves convergence performance of the FBPM filter, and generates an appropriate reference signal $x(n)$ for FxLMS based ANC adaptive filter $W(z)$. There are, however, a few problems as identified below:

1) It can work only if the reference signal is a narrowband signal.
2) The performance of this method depend on the proper choice the decorrelation delay $\Delta$.
3) In the case of a broadband noise source, the signal discrimination filter will not be able to predict it, and hence convergence of FBPM filter will be same as in previous method by Kuo et. al. [7]. Furthermore, the reference signal $x(n)$ will be highly corrupted, and it will degrade the convergence of $W(z)$.

C. Authors’ Previous Method

As shown in Fig. 6, this method uses an adaptive noise cancelation (ADNC) filter to improve the desired response for online FBPM filter. As in Kuo’s methods, a random noise signal, $v(n)$, uncorrelated with the reference noise, is added

\footnote{The same white signal can also be used for simultaneous online modeling of secondary path $S(z)$ [1]. In this paper, however, we assume that secondary path is exactly identified and $S(z) = S(z)$.

The error signal $e(n)$ is thus given as

$$e(n) = [d(n) - y_s(n)] - v_s(n) \quad (10)$$

with the output $y(n)$ of $W(z)$. The sum $y(n) + v(n)$ is propagated through the secondary loudspeaker. The residual error signal $e(n)$ is thus given as

$$u(n) = [h(n) * y(n) - v(n)] + e(n) \quad (11)$$

where $\mu_w$ is the step size for $W(z)$, $x'(n) = [x'(n), x'(n-1), \ldots, x'(n-L_w+1)]^T$, and $x'(n)$ is the reference signal $x(n)$ filtered through the secondary path modeling filter $\hat{S}(z)$. Assuming that $\hat{S}(z)$ is represented by an FIR filter of taps-length $L_s$, the filtered-reference signal $x'(n)$ is obtained as

$$x'(n) = \hat{s}^T(n)x_{L_s}(n) \quad (12)$$

where $\hat{s}(n) = [\hat{s}_0(n), \hat{s}_1(n), \ldots, \hat{s}_{L_h-1}(n)]^T$ is the impulse response of the $\hat{S}(z)$ and $x_{L_s}(n) = [x(n), x(n-1), \ldots, x(n-L_s+1)]^T$ is the $L_s$-sample reference signal vector.

The ADNC filter $H(z)$ takes $y(n)$ as its input, and $s'(n)$ as its desired response. The error signal of $H(z)$ is given as:

$$z(n) = [r(n) + y_f(n) - \hat{y}_f(n)] - u(n) + v_f(n) \quad (13)$$

where $u(n)$ is the output of ADNC filter $H(z)$, give as

$$u(n) = h^T(n)y_{L_h}(n) \quad (14)$$

where $h(n) = [h_0(n), h_1(n), \ldots, h_{L_h-1}(n)]^T$ is the impulse response of $H(z)$, $L_h$ is the taps-length of $H(z)$, and
\[ y_{L_h}(n) = [y(n-1), y(n-2), \cdots, y(n-L_h)]^T \] is the \( L_h \)-sample output signal vector. The coefficients of \( H(z) \) are updated by LMS algorithm:

\[ h(n+1) = h(n) + \mu_h z(n) y_{L_h}(n) \tag{15} \]

where \( \mu_h \) is step size parameter for \( H(z) \).

The adaptive FBPM filter \( \hat{F}(z) \) is excited by random (white) noise \( v(n) \), and its output is an estimate of feedback component \( v_f(n) \):

\[ \hat{y}_f(n) = \hat{f}^T(n) v(n) \tag{16} \]

where \( v(n) = [v(n), v(n-1), \cdots, v(n-L_f+1)]^T \) is the modeling noise vector. The error signal of \( H(z) \), \( z(n) \), is used as a desired response for adaptive FBPM filter \( \hat{F}(z) \) and its coefficients are updated by LMS algorithm:

\[ \hat{f}(n+1) = \hat{f}(n) + \mu_f (n) g(n) v(n) \tag{17} \]

where \( \mu_f(n) \) is step size parameter for \( \hat{F}(z) \), and \( g(n) = z(n) - \hat{y}_f(n) \). Here note the time dependence of the step size \( \mu_f(n) \), it will be explained later. After updating the tap-weights are copied to the fixed FBPM filter \( \hat{F}(z) \), taking \( y(n) \) as its input.

The reference signal \( x(n) \) for \( W(z) \) is computed by subtracting \( \hat{y}_f(n) \) from \( s'(n) \), and is given as

\[ x(n) = r(n) + [y_f(n) - \hat{y}_f(n)] + [v_f(n) - \hat{v}_f(n)]. \tag{18} \]

Assuming that \( H(z) \) is converged, \( u(n) \rightarrow r(n) + [y_f(n) - \hat{y}_f(n)] \approx z(n) \approx v_f(n) \). Thus adaptive FBPM filter \( \hat{F}(z) \) receives a desired response \( z(n) \) free of any disturbance. When \( \hat{F}(z) \) converges, i.e. \( \hat{F}(z) \approx F(z) \), then (ideally), \( \hat{y}_f(n) \approx y_f(n) \) and \( \hat{v}_f(n) \approx v_f(n) \). Thus the reference signal for the ANC filter \( W(z) \) is given as \( x(n) \approx r(n) \), and is free of any acoustic feedback component of the canceling signal.

In comparison with the Kuo’s method (Fig. 5) [8], the main advantage of the proposed method is that it can work with both narrowband and broadband noise sources. The reason is that it does not use prediction error filter and one does not need to consider correlation between \( s'(n) \) and \( s'(n-\Delta) \). Even in case of narrowband noise sources the Kuo’s method may fall behind the proposed method, as its performance depends on the proper choice of the decorrelation delay \( \Delta \).

\section{Proposed Method}

In (17), the error signal for \( \hat{F}(z) \), \( g(n) \) is given as:

\[ g(n) = z(n) - \hat{v}_f(n) = [r(n) + y_f(n) - \hat{y}_f(n)] - u(n) + [v_f(n) - \hat{v}_f(n)]. \tag{19} \]

Here \( [r(n) + y_f(n) - \hat{y}_f(n)] - u(n) \) acts as a disturbance signal for \( \hat{F}(z) \). Initially this disturbance signal is very large. When \( H(z) \) converges, \( u(n) \rightarrow r(n) + [y_f(n) - \hat{y}_f(n)] \) and this disturbance converges to (ideally) zero. This observation suggests that initially we should use a small value for the step size parameter \( \mu_f \), and later, when the disturbance signal is reduced, the step size can be increased accordingly. On the basis of \( z(n) \) and \( g(n) \), we use following indirect procedure to vary the step size \( \mu_f(n) \).

Define \( \rho(n) = P_g(n)/P_z(n) \) where \( P_g(n) \) and \( P_z(n) \) indicate the power of \( g(n) \) and \( z(n) \), respectively. These powers can be estimated as:

\[ P_g(n) = \lambda P_g(n-1) + (1-\lambda) \gamma^2(n) \tag{20} \]
where $\lambda$ is the forgetting factor ($0.9 < \lambda < 1$). From (13) and (19), the ratio $\rho(n)$ can be expressed as:

$$\rho(n) = \frac{P_g(n)}{P_z(n)} = \frac{P_{r(n)+y_f(n)-\hat{y}_f(n)-u(n)} + P_{v_f(n)}}{P_{r(n)+y_f(n)-\hat{y}_f(n)} - u(n) + P_{v_f(n)}}. \quad (21)$$

As stated earlier, the injected random noise $v(n)$ is a low level constant excitation signal. Initially, when the ANC system is started at $n = 0$, $u(n) = 0$, $\hat{y}_f(n) = 0$, $\hat{v}_f(n) = 0$, and $P_{r(n)+y_f(n)} \gg P_{v_f(n)} \Rightarrow \rho(n) \approx 1$. In steady state, as $n \to \infty$, $u(n) \to r(n) + [y_f(n) - \hat{y}_f(n)]$ and $\hat{v}_f(n) \to v_f(n)$, thus the expression in the numerator in (21) converges to zero, whereas the denominator is non-zero due to $P_{v_f(n)}$, and hence $\lim_{n\to\infty} \rho(n) \to 0$. This observation leads to the following mechanism for step size selection for $F(z)$:

$$\mu_f(n) = \rho(n)\mu_{f_{\text{min}}} + (1 - \rho(n))\mu_{f_{\text{max}}} \quad (22)$$

where $\mu_{f_{\text{min}}}$ and $\mu_{f_{\text{max}}}$ are the experimentally determined values for lower and upper bounds of the step size. These values are selected so that neither adaptation is too slow nor it becomes unstable. The proposed method is shown in shown in Fig. 7.

### III. Simulations

This section presents the simulation experiments performed to verify the effectiveness of the proposed method. The performance comparison is done on the basis of following performance measures.

- The estimation error for $F(z)$:
  $$\Delta F(\text{dB}) = 10\log_{10} \left\{ \frac{\sum_{i=0}^{M-1} [f_i(n) - \hat{f}_i(n)]^2}{\sum_{i=0}^{M-1} [f_i(n)]^2} \right\} \quad (23)$$

- The estimation error for $W(z)$:
  $$\Delta W(\text{dB}) = 10\log_{10} \left\{ \frac{\sum_{i=0}^{M-1} [w_i^*(n) - w_i(n)]^2}{\sum_{i=0}^{M-1} [w_i^*(n)]^2} \right\} \quad (24)$$

where $w_i(n)$ is the optimal value of the weight vector of the ANC controller. This is obtained by using the FxLMS algorithm under no acoustic feedback condition.

For acoustic paths the experimental data provided by [1] is used. Using this data, $P(z)$, $S(z)$ and $F(z)$ are selected as FIR filters of tap-weight lengths 48, 16, and 32, respectively. The frequency response of these paths is shown in Fig. 8. The control filter $W(z)$, FBPM filter $\hat{F}(z)$, and ADNC filter $H(z)$ are FIR filters of tap-weight length $L_w = 32$, $L_f = 32$, and $L_h = 16$, respectively. All the adaptive filters are initialized by null vectors of an appropriate order. A sampling frequency of 2 kHz is used. All the curves shown below are averaged over 10 experiments.

The reference signal $r(n)$ is a sinusoid of 300 Hz and a zero-mean white noise is added with SNR of 30 dB. The modeling excitation signal $v(n)$ is a zero-mean Gaussian noise of variance 0.05. The step size parameters are adjusted for fast and stable convergence, and, by trial-and-error, are found...
to be: $\mu_w = 1 \times 10^{-6}$, $\mu_f = 5 \times 10^{-3}$, $\mu_h = 5 \times 10^{-4}$, $\mu_{\text{f min}} = 5 \times 10^{-3}$, $\mu_{\text{f max}} = 1 \times 10^{-2}$. The decorrelation delay $\Delta$ in Kuo’s method [8] is 30.

The simulation results are shown in Fig. 9–11. Fig. 9 shows the curves for relative modeling error $\Delta F (\text{dB})$ for proposed method in comparison with the Kuo’s method, and authors’ previous method. We see that the proposed method can reduce FBPM error, $\Delta F$, at a faster convergence speed than the other methods. The corresponding curves for $\Delta W (\text{dB})$ are shown in Fig. 10. We see that the acoustic feedback degrades the performance of the FxLMS ANC system. This is in agreement with (2), which shows that in the presence of the acoustic feedback $W(z)$ may fail to converge to the optimal solution. The Kuo’s method and authors’ methods can reduce the effect of the acoustic feedback and hence, can reduce the estimation error $\Delta W$ to a lower level. Again, the proposed method gives best performance than the other methods. The variation of step size for the FBPM filter, $f(n)$, in the proposed method is shown in Fig. 11. It is seen that initially step size is adjusted to minimum value, $f_{\text{min}}$. Later, when $H(z)$ converges, the step size in increased to the maximum value $f_{\text{max}}$.

IV. CONCLUDING REMARKS

The presence of strong acoustic feedback degrades the convergence speed of the active noise control (ANC) filter, and in the worst case the ANC system may become unstable. A fixed feedback neutralization filter, obtained offline, can be used to neutralize the acoustic feedback. The feedback path, however, may be time varying, and we may need continual adjustments during online operation of the ANC system. Authors have proposed a method [5] for online FBPM in ANC systems, which uses an adaptive noise cancelation (ADNC) filter to remove disturbance in the desired response of FBPM filter. This improves the convergence of the FBPM filter, and hence overall performance of the ANC system. To further improve the convergence speed, here we incorporate a VSS LMS algorithm with the FBPM filter. It is demonstrated by computer simulations that the proposed method gives best performance among the existing methods. In future, it will be interesting to performance theoretical analysis of the proposed method. It will also be interesting to study the performance of the proposed method for some experimental setup.

REFERENCES