[N309] Feedforward Active Noise Control Systems with Online Secondary Path Modeling

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ABSTRACT

The basic approach for active noise control (ANC) systems with online secondary path modeling is called the additive random noise technique. It involves the injection of additional random noise into the ANC system and utilizes a system identification method to model the secondary path. This technique, proposed by Eriksson et al. (1989), comprises two processes; a noise-control process and a secondary-path modeling process. Improvements in this method have been proposed by many researchers, viz. Bao et al. (1993), Kuo et al. (1997), and Zhang et al. (2001). These improved methods introduce another adaptive filter into the ANC system, and hence design complexity is increased.

In this paper a new structure for feedforward ANC systems with online secondary-path modeling is proposed. The proposed structure 1) uses the same error signal for updating the noise-control process as used for the secondary-path modeling process, and 2) incorporates an averaging based filtered-reference algorithm in the noise-control process. In contrast to existing improved methods, no extra adaptive filter is introduced into the ANC system. The theoretical considerations are confirmed by computer simulations, which show that the proposed structure achieves better performance than that of the existing methods. This improved performance is achieved at the expense of a slightly increased computational complexity.

KEYWORDS: Active noise control, Averaging, FxLMS algorithm, Online secondary path modeling
EXISTINGS METHODS

The basic additive random noise technique for online secondary-path modeling in active noise control (ANC) [1] systems is proposed by Eriksson et al. [2]. As shown in Fig. 1, this ANC system comprises two processes; a noise-control process, and a secondary-path modeling process. Assuming that the control filter $W(z)$ is an FIR filter of order $L$, the secondary signal $y(n)$ is expressed as

$$y(n) = w^T(n)x_L(n)$$  \hspace{1cm} (1)$$

where $w(n)=[w_0(n) \, w_1(n) \, \ldots \, w_{L-1}(n)]^T$ and $x_L(n)=[x(n) \, x(n-1) \, \ldots \, x(n-L+1)]^T$ are tap weight vector and reference signal vector, respectively. An internally generated zero mean white Gaussian noise signal, $v(n)$, uncorrelated with the reference noise $x(n)$, is injected at the output $y(n)$ of the control filter. The residual noise signal $e(n)$ is given as

$$e(n) = d(n) - y'(n) + v'(n)$$  \hspace{1cm} (2)$$

where $d(n) = p(n) \ast x(n)$ is the primary disturbance signal at the error microphone, $y'(n) = s(n) \ast y(n)$ is the secondary canceling signal, $v'(n) = s(n) \ast v(n)$, $\ast$ denotes the convolution operation, and $p(n)$ and $s(n)$ are the impulse responses of the primary path $P(z)$ and secondary path $S(z)$, respectively. The residual noise signal $e(n)$ is used as an error signal for the control process, and as a desired response for the modeling process, i.e., $e_w(n) = d_s(n) = e(n)$. The coefficients of the control filter $W(z)$ are updated by the FxlMS algorithm:

$$w(n+1) = w(n) + \mu_w e_w(n)x'(n) = w(n) + \mu_w x'(n)u(n) + \mu_w x'(n)v'(n)$$  \hspace{1cm} (3)$$

where $\mu_w$ is the step size for the control process, $x'(n)$ is the reference signal $x(n)$ filtered through the modeling filter $\hat{S}(z)$, and $u(n) = d(n) - y'(n)$ is a component of the error signal due to canceling noise only. We see that the control process is perturbed by an undesired term $\mu_w x'(n)v'(n)$. Assuming that $\hat{S}(z)$ is represented by an FIR filter of order $M$, the filtered-reference signal $x'(n)$ is obtained as

$$x'(n) = \hat{s}^T(n)x_M(n)$$  \hspace{1cm} (4)$$
where \( \hat{s}(n) = [\hat{s}_0(n) \; \hat{s}_1(n) \; \cdots \; \hat{s}_{M-1}(n)]^T \) is the impulse response of the modeling filter \( \hat{S}(z) \) and \( \mathbf{x}_M(n) = [x(n) \; x(n-1) \; \cdots \; x(n-M+1)]^T \). The LMS update equation for \( \hat{S}(z) \) is given as

\[
\hat{s}(n+1) = \hat{s}(n) + \mu_s e_s(n)v(n) = \hat{s}(n) + \mu_s v(n)[d_s(n) - \hat{v}'(n)] \\
= \hat{s}(n) + \mu_s v(n)[\hat{v}'(n) - \hat{v}'(n)] + \mu_s v(n)u(n)
\]

where \( \mu_s \) is the step size of the modeling process, \( \hat{v}'(n) \) is an estimate of \( v'(n) \) obtained from the modeling filter, and \( v(n) = [v(n) \; v(n-1) \; \cdots \; v(n-M+1)]^T \). Eq. (5) shows that the performance of the modeling process is degraded by an undesired term \( \mu_s v(n)u(n) \) and in the worst case the modeling process may diverge.

Bao’s method is an attempt to improve the convergence of \( \hat{S}(z) \) in the presence of \( u(n) \), by introduction of adaptive noise cancellation (ADNC) filter (Fig. 2(a)) [3]. The ADNC filter

Fig. 1. Block diagram of active noise control systems with online secondary path modeling (Eriksson’s method).

Fig. 2. Improved methods for online secondary-path modeling in ANC system of Fig. 1: (a) Bao’s method (b) Kuo’s method, (c) Zhang’s method.
\( B(z) \), excited by \( x(n) \), cancels the interference introduced by \( u(n) \) in the estimation of \( S(z) \). In the method proposed by Kuo et al. (Fig. 2(b)), the adaptive prediction-error filter excited by delayed version of \( e(n) \), predicts and hence cancels the interference caused by \( u(n) \) [4]. In Zhang’s method the ADNC filter, \( W(z) \) and \( \hat{S}(z) \) are cross-updated to reduce the mutual coupling between the modeling process and the control process (Fig. 2(c)) [5]. The simulations conducted by Zhang et al. show that this structure gives the best performance of all existing methods. Hereafter these three methods are referred as improved methods.

**PROPOSED METHOD**

From above discussion we see that all improved methods concentrate on the modeling process and control process is same as in the Eriksson’s method. In present study we consider improving the performance of the control process. The basic idea is that if control process is efficient in its noise-reduction performance, then modeling process will in turn converge fast.

As shown in Fig. 1, the error signals for the control process and the modeling process are given as

\[
\begin{align*}
e_w(n) &= e(n) = u(n) + v'(n). \quad (6) \\
e_s(n) &= e(n) - \hat{v}'(n) = u(n) + [v'(n) - \hat{v}'(n)]. \quad (7)
\end{align*}
\]

From these expressions we observe that:

- Both \( e_w(n) \) and \( e_s(n) \) contain \( u(n) \), the error signal required for the noise control process.
- In \( e_w(n) \), \( u(n) \) is corrupted by the component \( v'(n) \) and in \( e_s(n) \) it is corrupted by the term \( [v'(n) - \hat{v}'(n)] \).
- As compared with \( e(n) \), \( e_s(n) \) is a better error signal for the control process, because \( |v'(n) - \hat{v}'(n)| < |v'(n)| \) and when \( \hat{S}(z) \) converges then (ideally) \( v'(n) \approx \hat{v}'(n) \Rightarrow [v'(n) - \hat{v}'(n)] \rightarrow 0 \).
- Since \( v(n) \) is a white Gaussian noise of zero mean hence both \( v'(n) \) and \( [v'(n) - \hat{v}'(n)] \) are random in nature and can be averaged out.

On the basis of the above analysis two modifications are suggested to the Eriksson’s method.
First, using $e_s(n)$ as an error signal for both the control process and the modeling process, i.e., $e_w(n) = e_s(n) = e(n) - \hat{v}(n)$. Second, replacing FxLMS algorithm with the adaptive filtering with averaging (AFA) [6] based filtered-x (FxAFA) algorithm. The proposed ANC system is shown Fig. 3.

The FxAFA algorithm [7] is a modified version of the FxLMS algorithm, where averaging is incorporated with both the iteration vector and the observation vector. Due to averaging the random fluctuations in the learning curves are reduced and hence iterations move towards the optimal solution at a faster rate. Referring to the FxLMS algorithm in Eq. (3), the FxAFA algorithm can be formulated as given in Eq. (8).

$$w(n+1) = \overline{w(n)} + \frac{1}{n^\gamma} \sum_{k=1}^{n} \mu \nu e_n(k) x'(k); \quad 1/2 < \gamma < 1$$

where $\overline{w(n)} = (1/n) \sum_{k=1}^{n} w(k)$. It is important to note that computing the running average of the data does not put so much computational burden since averages can be calculated recursively [7]. Now better noise reduction performance is expected and ANC will reduce the residual noise component $u(n)$ at fast convergence rate. This means that the modeling process is expected to converge fast now.

In comparison to existing improved methods, the main features of the proposed ANC system are summarized below:

- An effort is made to improve the control process, so that better noise reduction is achieved and hence modeling process converges fast.
- The proposed structure uses two adaptive filters, $W(z)$ and $\hat{S}(z)$, to perform noise-control and secondary-path modeling simultaneously. This is in contrast to existing improved methods, which use three adaptive filters.

**COMPUTATIONAL COMPLEXITY**

Table 1 presents the computational complexity (multiplications per iteration) comparison of the proposed methods with the existing methods. It is assumed that three adaptive filters, $B(z)$, $C(z)$ and $H(z)$ in Bao's method, Kuo's method and Zhang's method, respectively, are selected of order $N$. Hence these improved methods have the same computational complexity. The order of the third filter, $N$, is usually selected less than $L$ (a thumb rule is $N < L/2$). If a large value for $N$ is selected then the convergence of the third filter is slow, and overall performance
of the ANC system is degraded. With this constraint, the analytical expressions in Table 1 show that the computational complexity of the proposed method is greater than the existing schemes. To get a clear picture, a few numerical examples are also presented in Table 1.

**SIMULATION RESULTS**

In this section we compare the performance of the proposed method with those of the Eriksson's method and Zhang's method. The performance comparison is done on the basis of the relative modeling error, \( RME = 10 \log_{} \frac{\|s(n) - \hat{s}(n)\|}{\|s(n)\|} \). For the acoustical paths \( P(z) \) and \( S(z) \) the experimental data provided by [1] is used, where both are modeled by IIR filters of order 25. We truncate the impulse responses of the acoustic paths to get FIR (order 64) representation. These truncated impulse responses are shown in Fig. 4. The modeling filter \( \hat{S}(z) \) and control filter \( W(z) \), respectively, are FIR filters of order 64. The ADNC filter \( H(z) \) in Zhang's method is selected as an FIR filter of order 32. All initial values \( w(0), \hat{s}(0), \) and \( h(0) \) are set to zero. The sampling frequency of 4kHz is used.

### Table 1. Computational Complexity Comparison of the Proposed Method with the Existing Methods

<table>
<thead>
<tr>
<th>Analytical Expression</th>
<th>( L=M=64, N=32 )</th>
<th>( L=M=128, N=64 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eriksson’s Method</td>
<td>( 2L+3M+2 )</td>
<td>322</td>
</tr>
<tr>
<td>Improved Methods</td>
<td>( 2L+3M+2N+3 )</td>
<td>387</td>
</tr>
<tr>
<td>Proposed Method</td>
<td>( 6L+3M+2 )</td>
<td>578</td>
</tr>
</tbody>
</table>

![Fig. 3. Proposed method for active noise control systems with online secondary path modeling.](image-url)
The reference noise signal is a sinusoid of frequency 300 Hz and variance 2. To have low steady state residual noise, a zero-mean white Gaussian noise of variance 0.05 is used in the modeling process. The parameters are adjusted for fast and stable convergence. In Fig. 5, are shown the curves of the relative modeling error, $RME$, for the proposed methods in comparison to the existing methods. Here Modified Eriksson method is Eriksson's method with $e_w(n) = e_s(n) = e(n) - \hat{v}(n)$. This structure is obtained by replacing the FxAFA algorithm with the FxLMS algorithm in the ANC system of Fig. 3. We see that the modified Eriksson method beats the Eriksson's method, and the proposed method gives the best result of all.

REFERENCES


![Fig. 4. Impulse responses of acoustic paths: (a) Impulse response of primary path $P(z)$, (b) Impulse response of secondary path $S(z)$.

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![Fig. 5. Relative modeling error RME versus iteration time n: (a) Eriksson’s method ($\mu_w=1\times10^{-5}, \mu_s=5\times10^{-3}$), (b) Modified Eriksson method ($\mu_w=2\times10^{-5}, \mu_s=7\times10^{-3}$), (c) Zhang’s method ($\mu_w=2\times10^{-5}, \mu_s=2\times10^{-3}, \mu_h=2\times10^{-3}$), (d) Proposed method ($\mu_w=2\times10^{-3}, \mu_s=2\times10^{-3}, \gamma=0.5$).]