Author(s): Akhtar M. Tahir, Masahide Abe, Masayuki Kawamata
Title: Averaging Based Adaptive Algorithm for Single-channel Feedforward Active Noise Control Systems
Conference: 第17回ディジタル信号処理シンポジウム講演論文集
Number: C7-3
Date: November, 2002
Averaging Based Adaptive Algorithm for Single-channel Feedforward Active Noise Control Systems

Akhtar M. Tahir, Masahide Abe and Masayuki Kawamata
Department of Electronics Engineering, Graduate School of Engineering, Tohoku University, Sendai 980-8579, JAPAN.
Email: akhtar@mk.ecei.tohoku.ac.jp

Abstract
In this paper an active noise control (ANC) algorithm is proposed. This algorithm is based on adaptive filtering with averaging (AFA) and uses a similar structure as that of FxLMS ANC system, so we call it FxAFA algorithm. The proposed algorithm uses averages of both data and correction term to find updated values of the tap weights of ANC controller. The computer simulations are conducted for single-channel feedforward ANC systems. It is shown that the proposed algorithm gives fast convergence and high noise reduction, as compared with FxLMS, for both broadband and narrowband noise signals.

1. Introduction
In contrast to passive noise control (PSN), where absorbing materials are used to “absorb” the unwanted noise signal, the active noise control (ANC) [1] is based on the simple principle of destructive interference of propagating acoustic waves. The concept that acoustic wave interference can be controlled to produce zones of quietness was first proposed by P. Lueg in 1936 for an analogue ANC system. However, success with the early analogue controllers was very limited and in the recent years powerful DSP devices have made possible the development of real time ANC system with wide range of applications including air conditioning ducts, cars, aircrafts, and so on [2].

The schematic diagram for single-channel feedforward ANC systems is shown in Figure 1. The objective of the adaptive controller \( W(z) \) is to generate an appropriate antinoise signal \( y(n) \) propagated by the secondary loudspeaker. This antinoise signal combines with the primary noise signal to create a zone of silence in the vicinity of error microphone. The error microphone measures the residual noise \( e(n) \), which is used by \( W(z) \) for its adaptation to minimize the sound pressure at error microphone. Here \( S(z) \) accounts for the model of the secondary path between the output \( y(n) \) of the controller and that of error microphone \( e(n) \). The secondary path \( S(z) \) comprises DAC, LPF, power amplifier, loud speaker, acoustic path from loud speaker to error microphone, error microphone, pre-amp, anti-aliasing filter, and ADC. The filtering of the reference signal \( x(n) \) through \( S(z) \) is demanded by the fact that the output \( y(n) \) of the adaptive filter is filtered through \( S(z) \) [3]. The scheme of Figure 1 works well for both broadband and narrowband noise signals, but it suffers from the drawback of undesired acoustic feedback from the secondary loudspeaker to the reference microphone, for which feedback neutralizing technique is used that requires modeling the acoustic feedback path.

The most popular adaptation algorithm used for ANC applications (both broadband & narrowband) is FxLMS, which is a modified version of LMS algorithm [3]. Although computationally simple, the slow convergence rate of FxLMS algorithm has motivated researchers to look for fast converging algorithm; viz., IIR filter based LMS algorithm called Filtered-u Recursive LMS (FuRLMS) [4], RLS based algorithms called Filtered-x RLS (FxRLS) [1] and Filtered-x Fast-Transversal-Filter (FxFTF) [5]. The potential instability of IIR filter structures and great computational demands of RLS based algorithms still make FxLMS a good choice for ANC applications. The need for a fast converging yet a computationally simple algorithm for ANC applications is the main motivation for the investigation conducted in this paper. Here we explore the realization of an ANC algorithm using adaptive filtering with averaging (AFA) [6]. The simulation results show that this averaging based algorithm, which we call filtered-x AFA (FxAFA) algorithm, provides better results for both broadband and narrowband ANC systems.

Figure 1. Schematic diagram of a single-channel feedforward ANC system.
In Section 2, the proposed FxAFA algorithm is explained in connection with FxLMS algorithm. Sections 3 details the computer experiments performed, and in Section 4 concluding remarks are presented.

2. FxAFA Algorithm

Figure 2 shows the block diagram for a feedforward ANC system employing FxLMS algorithm for adaptation of the controller \( W(z) \). Here \( P(z) \) accounts for the primary acoustic path between the reference noise source and error microphone. \( S(z) \) is obtained offline and kept fixed during the online operation of ANC. The expression for the residual error \( e(n) \) is given as

\[
e(n) = d(n) - y'(n) \tag{1}
\]

where \( y'(n) \) is the controller output \( y(n) \) filtered through the secondary path \( S(z) \). Both \( y'(n) \) and \( y(n) \) are given as

\[
y'(n) = S^\dagger(n) y(n) \tag{2}
\]

\[
y(n) = w^\dagger(n) x(n) \tag{3}
\]

where \( w(n)=[w_0(n) \; w_1(n) \; \ldots \; w_{L-1}(n)]^T \) is the tap weight vector, \( x(n)=[x(n) \; x(n-1) \; \ldots \; x(n-L+1)]^T \) is the reference signal picked by the reference microphone and \( s(n) \) is the impulse response of the secondary path \( S(z) \). It is assumed that there is no acoustic feedback from the secondary loudspeaker to the reference microphone. The FxLMS update equation for the coefficients of \( W(z) \) is given as

\[
w(n+1) = w(n) + \mu e(n) x'(n) \tag{4}
\]

where \( x'(n) \) is the reference signal \( x(n) \) filtered through the secondary path model \( S(z) \):

\[
x'(n) = S^\dagger(n) x(n). \tag{5}
\]

Following the discussion in Ref. [6], the averaged algorithm corresponding to Eq. (4) can be formulated as

\[
w(n+1) = \overline{w(n)} + \mu \overline{e(n)x'(n)} \tag{6}
\]

where

\[
\overline{w(n)} = \frac{1}{n} \sum_{k=1}^{n} w(k) \tag{7}
\]

\[
\mu \overline{e(n)x'(n)} = \frac{1}{n'} \sum_{k=1}^{n'} \mu e(k) x'(k). \tag{8}
\]

In Ref. [6] the recommended value for \( \gamma \) is in the range \( 0.5 < \gamma < 1 \). It is important to note that computing the running averages of the data does not put so much computational burden since averages can be calculated recursively. For example, Eq. (7) can be recursively computed as

\[
\overline{w(n)} = \frac{1}{n} \left( (n-1) \overline{w(n-1)} + w(n) \right). \tag{9}
\]

Similarly Eq. (8) can be computed as

\[
\mu \overline{e(n)x'(n)} = \frac{1}{n'} \left( (n-1) \mu e(n-1) x'(n-1) + \mu e(n) x'(n) \right). \tag{10}
\]

Table 1. Computational complexity comparison between FxLMS and FxAFA

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Operation</th>
<th>Computations</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>FxLMS</td>
<td>Eq. (3)</td>
<td>A=L; M=L</td>
<td>3L+1</td>
</tr>
<tr>
<td></td>
<td>Eq. (5)</td>
<td>A=L; M=L</td>
<td>3L+1</td>
</tr>
<tr>
<td></td>
<td>Eq. (4)</td>
<td>A=L; M=L+1</td>
<td>3L+2</td>
</tr>
<tr>
<td></td>
<td>Eq. (6)</td>
<td>A=L</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Eq. (9)</td>
<td>A=L; D=1</td>
<td>2L+1</td>
</tr>
<tr>
<td></td>
<td>Eq. (10)</td>
<td>A=L+1; D=1</td>
<td>2L+2</td>
</tr>
</tbody>
</table>

\( M=\) Multiplications; \( D=\) Divisions; \( A=\) Additions; \( P=\) Power computations

Referring to the digital controllers marked in Figure 1, Eqs. (3), (5), (6), (9), and (10) are combined to give the proposed FxAFA algorithm. In Table 1, the computational complexity of FxLMS and FxAFA is compared on the basis of computations required for completion of operations per iteration. In this analysis both ANC controller \( W(z) \) and the secondary path model \( S(z) \) are assumed to be FIR filters of length \( L \). It is seen that the computational burden of the proposed FxAFA algorithm (7L+2 multiplications/iteration) is greater than that of FxLMS algorithm (3L+1 multiplications/iteration), but it is far less than that of FxRLS algorithm (3L+6L+2 multiplications/iteration [5]).

3. Computer Experiments

In this section the performance of the proposed FxAFA algorithm is demonstrated using computer simulation. The performance of the two algorithms (FxLMS and proposed FxAFA) is compared on the basis of noise reduction \( R \) (in dBs) [7] as follows:

\[
R = -10 \log_{10} \left( \frac{\sum e'(n)}{\sum d'(n)} \right) \tag{11}
\]

The large positive value of \( R \) indicates that more noise reduction is achieved. The simulations are carried out for two single-channel feedforward ANC setups. In the first experimental setup the primary acoustical path \( P(z) \) and the secondary path \( S(z) \) are modeled by IIR filters of order 25 (the data is provided on a disk included with [1]). The data for the second model is adopted from Ref. [8], where authors have identified \( P(z) \) and \( S(z) \) for laboratory scale experimental duct setup. The impulse responses of the primary and secondary paths for two setups are shown in Figure 3 and 4, respectively.
The secondary path model $\hat{S}(z)$ is an FIR filter of order 128 for setup 1 and of order 256 for setup 2, and is identified offline. Since industrial noise often has significant power in the frequency range between 50–250Hz [9], all simulations are carried with signals having frequency falling in this range. The sampling frequency of 2kHz is used. It is important to note that authors of Ref. [8] conducted the experiments at sampling frequency of 4kHz, so their data is appropriately modified to work for 2kHz. The parameters for the two algorithms (FxLMS and the proposed FxAFA) are adjusted for fast convergence.

3.1 Experiments with Broadband Signals

Broadband noise distributes its energy more or less evenly across the frequency band. Examples are low-frequency sounds of jet planes or impulse noise of an explosion [1]. In this section, some illustrative results of comparison between FxLMS and FxAFA are presented for broadband noise signals.

3.1.1 Case 1

The acoustic paths are of setup 1, as described previously. Three experiments are conducted to study performance and robustness of the proposed algorithm. In Experiment 1, the reference noise is a sinususoid containing five harmonics (of equal power) with the fundamental frequency of 50Hz. The white Gaussian noise of zero mean and variance 0.05 is added for any measurement noise present. This signal is used with the feedforward ANC system of Figure 2. The parameters for the two algorithms are adjusted to $\mu_{\text{FxLMS}}=10^{-5}$, $\mu_{\text{FxAFA}}=10^{-3}$, and $\gamma=0.6$. The performance, $R$, of the two algorithms for noise reduction is compared in Figure 5. We see that FxAFA algorithm gives 7dB more noise reduction and at faster convergence rate.

In Experiment 2, the reference noise is same as that of the first experiment, with the only difference that at time $t=4$sec the noise starts increasing (exponentially and slowly) and then settles at twice of the original value at $t=5.5$sec. The same parameters, as found in Case 1, are used for both algorithms. The performance, $R$, of the two algorithms for noise reduction is compared in Figure 6.

It is seen that the proposed FxAFA algorithm gives better performance.

In Experiment 3, the effect of any burst noise acquired by the error microphone is simulated. The additive white Gaussian noise of the first experiment is considered with unit variance from $t=4$sec to $t=4.25$sec (simulating a short burst). The performance, $R$, of the two algorithms for noise reduction is compared in Figure 7. Although two algorithms fall short of the performance achieved in Figure 5, still FxAFA gives 5dB more noise reduction.
3.1.2 Case 2

Here three experiments of Case 1 are repeated for acoustic paths of setup 2. The objective is to study the sensitivity of the parameters of the proposed algorithm with change of environment. The parameters for the two algorithms are adjusted to $\mu_{\text{FxLMS}}=10^{-7}$, $\mu_{\text{FxAFA}}=10^{-5}$, and $\gamma=0.6$. The results for three experiments are presented in Figures 8, 9 and 10, respectively. It is observed that 1) optimal parameters for two algorithms are found with slight tuning and 2) similar trend is observed in the performance of two algorithms as compared with experiments of Case 1.

3.2 Experiments with Narrowband Signals

For many applications, the primary noise is produced by rotating or reciprocating machines and is periodic (or nearly periodic) [1]. Here comparative results of FxLMS and FxAFA are presented for narrowband noise.

3.2.1 Case 3

Here simulations for setup 1 are performed for narrowband noise signal. In Experiment 1, a 200Hz sinusoidal signal with additive white Gaussian noise of variance 0.05 is considered. Parameters are adjusted for faster convergence and, for the two algorithms are found to be $\mu_{\text{FxLMS}}=10^{-4}$, $\mu_{\text{FxAFA}}=0.01$, and $\gamma=0.6$. Figure 11 shows the curves for noise reduction $R$ achieved by two algorithms. It is evident that the proposed FxAFA algorithm achieves 7dB more noise reduction at fast converging rate.

In Experiment 2, a 160Hz (randomly selected from 50–250Hz) sinusoid, simulating the noise generated by any other neighboring machinery, is added to the signal of 200Hz. For the parameters given in the last experiment, the noise reduction $R$ is compared in Figure 12. We see that FxAFA still gives better performance.

In Experiment 3, we consider the situation where the noise signal frequency is variable. The frequency of the signal (in Hz) is given as

$$f = \begin{cases} 
200, & 0 \leq t \leq 1.272 \text{sec} \\
208 - 8 \sin(-200 + 0.4t), & 1.272 < t \leq 9.127 \text{sec} \\
216, & 9.127 < t \leq 10 \text{sec}.
\end{cases}$$

The white Gaussian noise of variance 0.05 is added to the sinusoidal signal of frequency $f$. Same parameters as found previously are used for two algorithms. The curves for noise reduction $R$ are shown in the Figure 13. It is observed that both algorithms show robust behavior, although they fall short of performance achieved in Figure 11. Moreover, FxAFA gives 5dB more noise reduction.
3.2.2 Case 4

Here three experiments of Case 3 are repeated for the acoustic paths of setup 2. In Experiment 1, the parameters for the two algorithms are adjusted to $\mu_{\text{FxLMS}}=10^{-4}$, $\mu_{\text{FxAFA}}=0.005$, and $\gamma=0.6$, and noise reduction achieved is shown in Figure 14. In Experiment 2, both algorithms were unstable with the above parameters and after little tuning the results of Figure 15 are obtained for $\mu_{\text{FxLMS}}=5 \times 10^{-5}$, $\mu_{\text{FxAFA}}=0.002$, and $\gamma=0.6$. In the 3rd experiment the FxLMS algorithm (with $\mu_{\text{FxLMS}}=10^{-4}$) is unstable and the FxAFA algorithm (with $\mu_{\text{FxAFA}}=0.005$ and $\gamma=0.6$) deviates from the performance obtained in Figure 14. Thus FxLMS fails to cope with the frequency variations, whereas FxAFA remains stable thus giving somewhat superior performance to that of the FxLMS algorithm. The step size parameter of FxLMS is decreased for stable performance, and hence stability is achieved at the price of slow convergence. The results of Figure 16 are obtained for $\mu_{\text{FxLMS}}=2 \times 10^{-5}$, $\mu_{\text{FxAFA}}=0.005$, and $\gamma=0.6$. 

Figure 11. Noise reduction achieved by FxLMS and FxAFA in Case 3, Experiment 1

Figure 12. Noise reduction achieved by FxLMS and FxAFA in Case 3, Experiment 2

Figure 13. Noise reduction achieved by FxLMS and FxAFA in Case 3, Experiment 3

Figure 14. Noise reduction achieved by FxLMS and FxAFA in Case 4, Experiment 1

Figure 15. Noise reduction achieved by FxLMS and FxAFA in Case 4, Experiment 2

Figure 16. Noise reduction achieved by FxLMS and FxAFA in Case 4, Experiment 3
3.2.3 Case 5

Here the performance of the two algorithms discussed in this paper is compared with that of FxRLS algorithm. Experiment 1 of Case 3 is repeated for the purpose and parameters found therein are used for FxLMS and FxAFA algorithms. The FxRLS algorithm is implemented according to the guidelines of Ref. [5]. The parameters for FxRLS algorithm are adjusted for fast and stable convergence and are found to be $\mu_{\text{FxRLS}} = 0.1$, $\lambda = 0.99$ and $\delta = 0.04$. To stabilize the FxRLS algorithm random noise is added in the algorithm to the input signal with SNR of 15dB (see Ref. [5] for details). The noise reduction curves are shown in Figure 17. It is seen that FxRLS is stable only for duration of 1.5sec and diverges very quickly after that. Adding large noise to the input of FxRLS algorithm or decreasing step size parameter can increase the stable region of operation but it slows down the convergence speed.

4. Conclusions

Here a new ANC algorithm based on adaptive filtering with averaging is presented. Computer simulations are conducted for single-channel feedforward ANC systems using both broadband and narrowband signals. Different case studies have been performed which demonstrate effectiveness of the proposed FxAFA algorithm. It is seen that the proposed algorithm achieves faster convergence and more residual noise reduction, but at the expense of slightly increased complexity. The robustness of the proposed algorithm is comparable to that of FxLMS. It has two adjustable parameters; $\gamma$ and $\mu$, and requires more care in selecting their values. It is seen that by proper choice of $\gamma$ the larger value for $\mu$ can be selected, and thus faster convergence can be achieved.

In order to observe the sensitivity of two algorithms with changing environment simulations are carried out for two different setups. It is seen that with little tuning both algorithms can be stabilized. An experiment is performed with FxRLS algorithm. This is a very limited experiment valid only for a particular signal and a particular system. Nevertheless it demonstrates the superiority of the proposed FxAFA algorithm over FxRLS algorithm in both computational complexity and stability.

In this paper offline secondary path modeling is used and it is assumed that secondary path remains fixed all the time. For some applications the secondary path may be time varying, and it is desirable to estimate the secondary path online when the ANC is in operation [10]. This is a system identification (SI) problem with very low SNR and averaging based adaptive algorithm has shown good performance in such SI cases. The development of an ANC algorithm with online secondary path modeling, incorporating adaptive filtering with averaging is task of future work.

5. References